

Numbers and Bases

September 08, 2000

CSC201 Section 002

Fall, 2000

Weighted Positional (Decimal) Notation

- Base = 10
- n-digit number written as $d_{n-1} d_{n-2} \dots d_0$
 - most significant digit = d_{n-1} , least significant digit = d_0
- Value of number = $\sum_{i=0..n-1} d_i * 10^i$
- Example:
 - $1093_{10} = 1 \times 10^3 + 0 \times 10^2 + 9 \times 10^1 + 3 \times 10^0$

$$1093_{10} = 1 \times 10^3 + 0 \times 10^2 + 9 \times 10^1 + 3 \times 10^0$$

Unsigned Binary (Base Two) Representation

- Base = 2
- n-bit number written as $b_{n-1} b_{n-2} \dots b_0$
 - most significant bit = b_{n-1} , least significant bit = b_0
- Value of number = $\sum_{i=0..n-1} b_i * 2^i$
- Powers of two

i	2^i
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512
10	1024

Example:

$1011_2 =$

$$1 * 2^3 + 0 * 2^2 +$$

$$1 * 2^1 + 1 * 2^0$$

Unsigned Hexadecimal (Base 16) Representation

- Base = 16
- n-digit number written as $h_{n-1} h_{n-2} \dots h_0$
 - most significant digit = h_{n-1} , least significant digit = h_0
- Value of number = $\sum_{i=0..n-1} h_i * 16^i$
- Powers of sixteen

i	16^i
1	16
2	256
3	4096
4	64K

- Hex digits = 0-9, A, B, C, D, E, and F

$$8C2A_{16} = 8 * 16^3 + 12 * 16^2 + 2 * 16^1 + 10 * 16^0$$

Converting Hex or Binary to Decimal

- Use the expressions shown above, and know the powers of 16 (hex) or powers of 2 (binary)

Converting Hex to/from Binary

- Replace each 4 bits with one hex digit, and vice versa

$$1110\ 0011\ 0111\ 0001_2 = E\ 3\ 7\ 1_{16}$$

Converting Decimal to Binary

- Divide by two repeatedly
- The remainder at each step is the next binary digit, from lsb to msb
- Stop when the quotient is 0

Example: convert 365 to binary

$$365 / 2 = 182, \text{ remainder (lsb)} = 1$$

$$182 / 2 = 91, \text{ remainder} = 0$$

$$91 / 2 = 45, \text{ remainder} = 1$$

$$45 / 2 = 22, \text{ remainder} = 1$$

$$22 / 2 = 11, \text{ remainder} = 0$$

$$11 / 2 = 5, \text{ remainder} = 1$$

$$5 / 2 = 2, \text{ remainder} = 1$$

$$2 / 2 = 1, \text{ remainder} = 0$$

$$1 / 2 = 0, \text{ remainder (msb)} = 1$$